Surname	Centre Number	Candidate Number
First name(s)		2

GCE AS/A LEVEL

S24-2305U10-1

2305U10-1

MONDAY, 13 MAY 2024 – AFTERNOON

FURTHER MATHEMATICS – AS unit 1
FURTHER PURE MATHEMATICS A

1 hour 30 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	5	
2	3	
3	6	
4	10	
5	7	
6	12	
7	7	
8	12	
9	8	
Total	70	

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 70.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. The complex numbers z, v and w are related by the equation

$$z = \frac{v}{w}.$$

Given that $v = -16 + 11\mathrm{i}$ and $w = 5 + 2\mathrm{i}$, find z in the form $r\left(\cos\theta + \mathrm{i}\sin\theta\right)$. [5]

Examiner only

2. Given that $x^2 + 4x + 5$ is a factor of $x^3 + x^2 - 7x - 15$, solve the equation $x^3 + x^2 - 7x - 15 = 0$.

[3]

Examiner only

3. The quadratic equation $x^2 + px + q = 0$ has a repeated root α.

A new quadratic equation has a repeated root $\frac{1}{\alpha}$ and is of the form $x^2 + mx + m = 0$.

Find the values of p and q in the original equation. [6]

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4. The complex numbers z and w are represented, respectively, by the points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = \frac{z}{1-z}\,.$$

(a) Show that $v = \dfrac{y}{(1-x)^2 + y^2}$ and obtain an expression for u in terms of x and y. [5]

Examiner only

(b) The point *P* moves along the line $y = 1 - x$. Find and simplify the equation of the locus of *Q*. [5]

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5. Given that

$$\sum_{r=k}^{76}(r-31)=980,$$

show that there are two possible values of k. [7]

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6. The complex number z is represented by the point $P(x, y)$ in an Argand diagram.

Two loci, L_1 and L_2, are given by:

$$L_1: \ |z-2+\mathrm{i}| = |z+2-3\mathrm{i}|,$$

$$L_2: \ |z-2+\mathrm{i}| = \sqrt{10}.$$

(a) Find the coordinates of the points of intersection of these loci. [9]

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(b) On the same Argand diagram, sketch the loci L_1 and L_2. Clearly label the coordinates of the points of intersection. [3]

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7. Prove, by mathematical induction, that $13^{(2n-1)}+8$ is a multiple of 7 for all positive integers n. [7]

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8. A point *P* is reflected in the line $y = kx$, where k is a constant. It is then rotated anticlockwise about *O* through an acute angle θ, where $\cos\theta = 0{\cdot}8$. The resulting transformation matrix is given by *T*, where

$$T = \frac{1}{85}\begin{bmatrix} -84 & -13 \\ -13 & 84 \end{bmatrix}.$$

(a) Determine the value of k. [9]

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(b) Find the invariant points of *T*. [3]

Examiner only

9. Two planes, Π_1 and Π_2, are defined by

$$\Pi_1 : 4x - 3y + 2z = 5,$$

$$\Pi_2 : 6x + y + z = 9.$$

(a) Find the acute angle between the planes Π_1 and Π_2. Give your answer correct to three significant figures. [4]

Examiner only

(b) Find the perpendicular distance from the point $A(5, -2, -6)$ to the plane Π_1. [2]

(c) (i) Show that the point $B(5, 5, 0)$ lies on Π_1 and that the point $C(1, 3, 0)$ lies on Π_2.

(ii) State an equation of a plane that contains the points B and C. [2]

END OF PAPER

Question number	Additional page, if required. Write the question number(s) in the left-hand margin.

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